

CP Asymmetry of $B \rightarrow X_s l^+ l^-$ in Low Invariant Mass RegionS. Fukae^{a1},^a*Department of Physical Science, Hiroshima University,
Higashi Hiroshima 739-8526, Japan***Abstract**

I analyzed the CP asymmetry of $B \rightarrow X_s l^+ l^-$ based on model-independent analysis which includes twelve independent four Fermi operators. The CP asymmetry is suppressed in the Standard Model, however, if some new physics make it much larger, the present or the next generation B factories may catch the CP violation in this decay mode. In this paper, we study the correlation of the asymmetry and the branching ratio, and then we will find only a type of interactions can be enlarge the asymmetry. Therefore, in comparison with experiments, we have possibility that we can constrain models beyond the standard model.

1 Introduction

The inclusive rare B decay $B \rightarrow X_s l^+ l^-$ has already been studied by many researchers. It is attractive to investigate this process experimentally or theoretically. This decay mode is experimentally clean as well as $B \rightarrow X_s \gamma$, specially in the low invariant mass region. And, when we can use a parton model to study this process theoretically, because it is semileptonic decay. In the standard model (SM), a flavor changing neutral current (FCNC) process appears only through one or more loops. Since $B \rightarrow X_s l^+ l^-$ is also a FCNC, new physics can clarify itself to measure this decay. The extended models beyond the SM like the minimal supersymmetrized model (MSSM) and the two Higgs doublets model (2HDM) predict some deviation from the SM[1] -[14]. The SM prediction shows that, for $l = e$ or μ , this mode will be found at the KEKB and the SLAC e^+e^- storage ring PEP-II B factories in near future. Therefore, the study of this process is one of the most interesting topic in order to search new physics. In this paper, the final leptons will be a μ ons or electrons throughly.

The CP-violating asymmetry of this decay is also a subject that many physicists investigate. This observable is very sensitive to the complex phase of the CKM matrix elements, so that we have the possibility to find effects beyond the SM. The SM predicts that the CP asymmetry is suppressed, about 10^{-3} or smaller[15, 16]. If some non-SM interactions enlarge for the asymmetry to get sizable, we can know the existence beyond SM. This observable has been calculated in MSSM and 2HDM[10] -[14]. In these models, as well as the SM, the distribution is a function of fewer Wilson coefficients than the full operator basis. In our previous work, we analyzed the branching ratio and the forward-backward (FB) asymmetry, which is an observable corresponding to the size of parity violation in the decay $B \rightarrow X_s l^+ l^-$, with a most general model-independent method[17, 18]. Generally, the matrix element for the decay $b \rightarrow s l^+ l^-$ includes all types of local and $bs\gamma$ -induced four-Fermi operators. That is,

$$\begin{aligned} \mathcal{M} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi} V_{ts}^* V_{tb} \quad [& C_{SL} \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_s L) b \bar{l} \gamma^\mu l \\ & + C_{BR} \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_b R) b \bar{l} \gamma^\mu l \\ & + C_{LL} \bar{s} \gamma_\mu b_L \bar{l} \gamma^\mu l_L \\ & + C_{LR} \bar{s} \gamma_\mu b_L \bar{l} \gamma^\mu l_R \end{aligned}$$

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$$\begin{aligned}
& + C_{RL} \bar{s}_R \gamma_\mu b_R \bar{l}_L \gamma^\mu l_L \\
& + C_{RR} \bar{s}_R \gamma_\mu b_R \bar{l}_R \gamma^\mu l_R \\
& + C_{LRLR} \bar{s}_L b_R \bar{l}_L l_R \\
& + C_{RLLR} \bar{s}_R b_L \bar{l}_L l_R \\
& + C_{LRRL} \bar{s}_L b_R \bar{l}_R l_L \\
& + C_{RLRL} \bar{s}_R b_L \bar{l}_R l_L \\
& + C_T \bar{s} \sigma_{\mu\nu} b \bar{l} \sigma^{\mu\nu} l \\
& + i C_{TE} \bar{s} \sigma_{\mu\nu} b \bar{l} \sigma_{\alpha\beta} l \epsilon^{\mu\nu\alpha\beta},
\end{aligned} \tag{1}$$

where C_{XX} 's are the coefficients of the four-Fermi interactions. Among them, there are two $bs\gamma$ induced four-Fermi interactions denoted by C_{SL} and C_{BR} , which correspond to $-2C_7^{eff}$ in the SM, and which are constrained by the experimental data of $b \rightarrow s\gamma$. There are four vector-type interactions denoted by C_{LL} , C_{LR} , C_{RL} , and C_{RR} . Two of them (C_{LL} , C_{LR}) are already present in the SM as the combinations of $(C_9 - C_{10}, C_9 + C_{10})$. Therefore, they are regarded as the sum of the contribution from the SM and the new physics deviations ($C_{LL}^{new}, C_{LR}^{new}$). The other vector interactions denoted by C_{RL} and C_{RR} are obtained by interchanging the chirality projections $L \leftrightarrow R$. There are four scalar-type interactions, C_{LRLR} , C_{RLLR} , C_{RLLR} and C_{RLRL} . The remaining two denoted by C_T and C_{TE} correspond to tensor-type. The indices, L and R , are chiral projections, $L = \frac{1}{2}(1 - \gamma_5)$ and $R = \frac{1}{2}(1 + \gamma_5)$. Then, we can get the differential branching ratio of the FCNC process $b \rightarrow sl^+l^-$,

$$\begin{aligned}
\frac{d\mathcal{B}}{ds} = \frac{1}{2m_b^8} \mathcal{B}_0 \text{Re}[& S_1(s) \{m_s^2 |C_{SL}|^2 + m_b^2 |C_{BR}|^2\} \\
& + S_2(s) \{2m_b m_s C_{SL} C_{BR}^*\} \\
& + S_3(s) \{2m_s^2 C_{SL} (C_{LL}^* + C_{LR}^*) + 2m_b m_s C_{BR} (C_{RL}^* + C_{RR}^*)\} \\
& + S_4(s) \{2m_b^2 C_{BR} (C_{LL}^* + C_{LR}^*) + 2m_b m_s C_{SL} (C_{RL}^* + C_{RR}^*)\} \\
& + M_2(s) \{|C_{LL}|^2 + |C_{LR}|^2 + |C_{RL}|^2 + |C_{RR}|^2\} \\
& + M_6(s) \{-2(C_{LL} C_{RL}^* + C_{LR} C_{RR}^*) \\
& \quad + (C_{LRLR} C_{RLLR}^* + C_{LRRL} C_{RLRL}^*)\} \\
& + M_8(s) \{|C_{LRLR}|^2 + |C_{RLLR}|^2 + |C_{LRRL}|^2 + |C_{RLRL}|^2\} \\
& + M_9(s) \{16|C_T|^2 + 64|C_{TE}|^2\}],
\end{aligned} \tag{2}$$

Here, we ignore terms including lepton mass m_l , because we take only massless (anti-) lepton into consideration. A set of the kinematic functions $S_i(s)$ ($i = 1, 2, 3, 4, 5, 6$) and $M_n(s)$ ($n = 2, 6, 8$) is shown in Appendix A. The normalization factor \mathcal{B}_0 is given by

$$\mathcal{B}_0 \equiv \mathcal{B}_{sl} \frac{3\alpha^2}{16\pi^2} \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{1}{f(\hat{m}_c) \kappa(\hat{m}_c)}, \tag{3}$$

where the other factors $f(\hat{m}_c)$ and $\kappa(\hat{m}_c)$ are the phase space factor and the $O(\alpha_s)$ QCD correction factor[19]. The factor \mathcal{B}_{sl} denotes the branching ratio of the semileptonic decay, and we set it to 10.4%. We can also have the FB asymmetry from Eq.(1). Thus, by numerical analysis, we got useful information to pin down new physics beyond the standard model. However, we set all the new Wilson coefficients to real when we carry out the numerical analysis. This means that we assume that there is no new cp-violating source in the decay $B \rightarrow X_s l^+ l^-$. The CP asymmetry is sensitive to the imaginary part of the coefficients. Therefore, it is worth treating the CP asymmetry based on the our previous analysis.

This paper is organized as follows. In Section 2, we find the way how to get the general CP asymmetry, study the correlation between the asymmetry and the branching ratio to pin down the type of interactions and give some discussions. We give summary in Section 3.

2 General CP Asymmetry

We assume semileptonic decay $b \rightarrow cl^+ \bar{l}$ is an approximately CP-conserving mode, in fact experiments shows they corresponds with each other within about 10^{-2} [20]. And, the partonic approximation predicts

no CP-violating asymmetry in the standard model (SM). That is, we can use the same normalization factor as Eq.(3) to express the branching ratio of $b \rightarrow sl^+l^-$ and $\bar{b} \rightarrow \bar{s}l^+l^-$. For a general Wilson coefficient C_{XX} , we can define B_{XX} , λ_{XX} and A_{XX} by

$$C_{XX} \equiv B_{XX} + \lambda_{XX} A_{XX}, \quad (4)$$

where λ_{XX} is CP violating phase and generally the both of B_{XX} and A_{XX} are complex. In the case of the SM, only the CKM matrix elements give the CP violating weak phase and the strong phase appears through QCD penguin correction. Conventionally, these effects are included in the Wilson coefficients C_9^{eff} of the vector-type current-current interaction[21]. Explicitly it is expressed by[21, 22]

$$C_9^{eff} = B_9 + \lambda_u A_9, \quad (5)$$

where, without $c\bar{c}$ long-distant contribution,

$$B_9 = \left(1 + \alpha \frac{w(s)}{\pi}\right) C_9^{NDR} + Y(s). \quad (6)$$

Only $\lambda_u \equiv (V_{ub}V_{us}^*)/(V_{tb}V_{ts}^*)$ includes CP-violating phase. Since $\lambda_u A_9$ is very small except for $c\bar{c}$ resonance region, the SM predicts that the CP asymmetry is very negligible[15].

We must take $c\bar{c}$ resonance into consideration to discuss the branching ratio and the CP asymmetry[23], otherwise avoid region where J/ψ and ψ' poles give contribution[16]. In this paper, we take the latter stand. The residual region is lower region before J/ψ resonance or higher region after ψ' resonance[13]. We restrict our discussion to only low invariant mass region, $1 < s < 8$ (GeV^2), where $s \equiv (p_{l^+} + p_{l^-})^2$. We then introduce the partially integrated CP asymmetry \mathcal{A}_{CP} defined by

$$\mathcal{A}_{CP} \equiv \frac{\mathcal{B}(B \rightarrow X_s l^+ l^-) - \mathcal{B}(\bar{B} \rightarrow X_s l^+ l^-)}{\mathcal{B}(B \rightarrow X_s l^+ l^-) + \mathcal{B}(\bar{B} \rightarrow X_s l^+ l^-)} \equiv \frac{\mathcal{N}_{CP}}{\mathcal{D}_{CP}}, \quad (7)$$

where $\mathcal{B}(B \rightarrow X_s l^+ l^-)$ is the partially integrated branching ratio for process $B \rightarrow X_s l^+ l^-$, defined by

$$\int_{1(\text{GeV}^2)}^8 ds \frac{d\mathcal{B}(B \rightarrow X_s l^+ l^-)}{ds} \sim 3.73 \times 10^{-6} \quad (\text{at } \mu = (m_b)_{\overline{MS}}).$$

In the same way, we define the partially integrated branching ratio for $\bar{B} \rightarrow X_s l^+ l^-$. We set $(C_7^{eff}, C_9^{NDR}, C_{10}) = (-0.317, 4.52, -4.29)$ for numerical calculation. We listed its value for the SM at renormalization scale $\mu = (m_b)_{\overline{MS}} = 4.2$ GeV in Table 1, where we set the Wolfenstein's CKM parameters[24] to $(\rho, \eta) = (0.12, 0.25)$, $(0.16, 0.33)$ and $(0.27, 0.40)$. We should note that there is the huge uncertainty about the

(ρ, η)	\mathcal{A}_{CP}^{SM}
(0.12, 0.25)	0.85×10^{-3}
(0.16, 0.33)	1.12×10^{-3}
(0.27, 0.40)	1.36×10^{-3}

Table 1: The partially integrated CP asymmetry for $(\rho, \eta) = (0.12, 0.25)$, $(0.16, 0.33)$ and $(0.27, 0.40)$ and in the SM at $\mu = (m_b)_{\overline{MS}}$.

CP Asymmetry predicted by the SM before we discuss the sensitivity to new physics from our numerical results. The asymmetry in the SM is uncertain by almost 100 % [16]. So, we must get at least 10 times large size as the SM prediction about the CP asymmetry to find the signal of new physics, otherwise we fail to do. Then, from Eq.(1), we can get the numerator \mathcal{N}_{CP} of the CP asymmetry by replacing $\text{Re}(C_{XX}C_{YY}^*)$ in the branching ratio given in Eq.(2) with

$$-2\text{Im}(\lambda_{XX})\text{Im}(B_{YY}^*A_{XX}) - 2\text{Im}(\lambda_{YY})\text{Im}(B_{XX}^*A_{YY}) - 2\text{Im}(\lambda_{XX}\lambda_{YY}^*)\text{Im}(A_{XX}A_{YY}^*),$$

and, for the dominator \mathcal{D}_{CP} , with

$$2\text{Re}(B_{XX}B_{YY}^*) + 2\text{Re}(\lambda_{XX})\text{Re}(B_{YY}^*A_{XX}) + 2\text{Re}(\lambda_{YY})\text{Re}(B_{XX}^*A_{YY}) + 2\text{Re}(\lambda_{XX}\lambda_{YY}^*)\text{Re}(A_{XX}A_{YY}^*).$$

The resultant CP asymmetry takes the most general model-independent form. We show the explicit expression of this asymmetry in Appendix A. The CP asymmetry does not vanish if and only if B_{XX} or A_{XX} has deferent phase from A_{YY} and λ_{YY} or $\lambda_{XX}\lambda_{YY}^*$ has a imaginary part. Here, XX and YY denote types of interactions, whether they are the same type or not. However, in the most interesting models like the two higgs doublet model (2HDM) [13, 14] and the minimal supersymmetrized standard model (MSSM)[1, 11], the strong phase does not play a so important role on the CP asymmetry. Therefore, we assume that we can ignore a set of strong phase introduced by new physics[9]. Then, for new vector, scalar and tensor-type interactions, we can redefine the Wilson coefficients as

$$C_{XX} = B_{XX}^{SM} + (\lambda_{XX} + \lambda_u)(A_9 + A_{XX}) \quad \text{for } XX = LL \text{ or } LR, \quad (8)$$

$$C_{XX} = \lambda_{XX}A_{XX} \quad \text{for others,} \quad (9)$$

Here, A_{XX} s are real and λ_{XX} s are phase factors defined by $\exp(i\phi_{XX})$ where $0 \leq \phi_{XX} < 2\pi$, and $B_{LL}^{SM} \equiv B_9 - C_{10}$ and $B_{LR}^{SM} \equiv B_9 + C_{10}$. In the same way, we can redefine C_{BR} and C_{SL} , and have other constraints from the measurement of $B \rightarrow X_s\gamma$,

$$4|C_7^{eff}|^2(m_b^2 + m_s^2) = m_b^2(|A_{SL}^N|^2 + |A_{BR}|^2), \quad (10)$$

where $A_{SL}^N = (m_b/m_s)A_{SL}$ [18]. The definitions of A_{BR} and A_{SL} , and ϕ_{BR} and ϕ_{SL} , follows Eq.(9). Thus if there is the interference between such coefficients and the C_9^{eff} , it can enlarge the CP asymmetry. Otherwise, the new interactions suppress the observable under the above assumption. In this case, the explicit form of the partially integrated CP asymmetry is given by

$$\mathcal{A}_{CP} \equiv \frac{\int_{1\text{GeV}^2}^8 ds (d\mathcal{N}_{CP}(s)/ds)}{\int_{1\text{GeV}^2}^8 ds (d\mathcal{D}_{CP}(s)/ds)} \equiv \frac{\mathcal{N}_{CP}}{\mathcal{D}_{CP}}, \quad (11)$$

where

$$\begin{aligned} \frac{d\mathcal{N}_{CP}(s)}{ds} = & -\frac{1}{m_b^8} \mathcal{B}_0 [\\ & S_3(s) \{ 2m_s^2 (Im(\lambda_{SL})Im(A_{SL}B_9^*) + Im(\lambda_{SL}\lambda_{LL}^*)Im(A_{SL}A_9^*) \\ & \quad + Im(\lambda_{SL})Im(A_{SL}B_9^*) + Im(\lambda_{SL}\lambda_{LR}^*)Im(A_{SL}A_9^*)) \} \\ & + S_4(s) \{ 2m_b^2 (Im(\lambda_{BR})Im(A_{BR}B_9^*) + Im(\lambda_{BR}\lambda_{LL}^*)Im(A_{BR}A_9^*) \\ & \quad + Im(\lambda_{BR})Im(A_{BR}B_9^*) + Im(\lambda_{BR}\lambda_{LR}^*)Im(A_{BR}A_9^*)) \} \\ & + M_2(s) \{ 2 (Im(\lambda_{LL})Im((B_9 - C_{10})(A_9 + A_{LL})^*) + Im(\lambda_{LR})Im((B_9 + C_{10})(A_9 + A_{LR})^*)) \} \\ & + M_6(s) \{ -2 (Im(\lambda_{RL})Im((B_9^* - C_{10})A_{RL}) + Im(\lambda_{LL}\lambda_{RL}^*)Im((A_9 + A_{LL})A_{RL}^*) \\ & \quad + (Im(\lambda_{RR})Im((B_9^* + C_{10})A_{RR}) + Im(\lambda_{LR}\lambda_{RR}^*)Im((A_9 + A_{LR})A_{RR}^*)) \} \}, \end{aligned} \quad (12)$$

and

$$\begin{aligned} \frac{d\mathcal{D}_{CP}(s)}{ds} = & \frac{1}{m_b^8} \mathcal{B}_0 [\\ & S_1(s) \{ m_s^2 |A_{SL}|^2 + m_b^2 |A_{BR}|^2 \} \\ & + S_2(s) \{ 2m_b m_s \text{Re}(\lambda_{SL}\lambda_{BR}^*) \text{Re}(A_{SL}A_{BR}^*) \} \\ & + S_3(s) \{ 2m_s^2 (\text{Re}(\lambda_{SL})\text{Re}(A_{SL}(B_9 - C_{10})^*) + \text{Re}(\lambda_{SL}\lambda_{LL}^*)\text{Re}(A_{SL}(A_9 + A_{LL})^*) \\ & \quad + \text{Re}(\lambda_{SL})\text{Re}(A_{SL}(B_9 + C_{10})^*) + \text{Re}(\lambda_{SL}\lambda_{LR}^*)\text{Re}(A_{SL}(A_9 + A_{LR})^*)) \\ & \quad + 2m_b m_s (\text{Re}(\lambda_{BR}\lambda_{RL}^*)\text{Re}(A_{BR}A_{RL}^*) + \text{Re}(\lambda_{BR}\lambda_{RR}^*)\text{Re}(A_{BR}A_{RR}^*)) \} \\ & + S_4(s) \{ 2m_b^2 (\text{Re}(\lambda_{BR})\text{Re}(A_{BR}(B_9 - C_{10})^*) + \text{Re}(\lambda_{BR}\lambda_{LL}^*)\text{Re}(A_{BR}(A_9 + A_{LL})^*) \\ & \quad + \text{Re}(\lambda_{BR})\text{Re}(A_{BR}(B_9 + C_{10})^*) + \text{Re}(\lambda_{BR}\lambda_{LR}^*)\text{Re}(A_{BR}(A_9 + A_{LR})^*)) \} \end{aligned}$$

$$\begin{aligned}
& +2m_b m_s (Re(\lambda_{SL}\lambda_{RL}^*)Re(A_{SL}A_{RL}^*) + Re(\lambda_{SL}\lambda_{RR}^*)Re(A_{SL}A_{RR}^*))\} \\
& +M_2(s) \{|B_9 - C_{10}|^2 + |A_9 + A_{LL}|^2 + 2Re(\lambda_{LL})Re((B_9 - C_{10})(A_9 + A_{LL})^*) \\
& \quad + |B_9 + C_{10}| + |A_9 + A_{LR}|^2 + 2Re(\lambda_{LR})Re((B_9 + C_{10})(A_9 + A_{LR})^*) \\
& \quad + |A_{RL}|^2 + |A_{RR}|^2\} \\
& +M_6(s) \{-2 (Re(\lambda_{RL})Re((B_9 - C_{10})^* A_{RL}) + Re(\lambda_{LL}\lambda_{RL}^*)Re((A_9 + A_{LL})A_{RL}^*) \\
& \quad + Re(\lambda_{RR})Re((B_9 + C_{10})^* A_{RR}) + Re(\lambda_{LR}\lambda_{RR}^*)Re((A_9 + A_{LR})A_{RR}^*)) \\
& \quad + (Re(\lambda_{LRLR}\lambda_{RLLR}^*)Re(A_{LRLR}A_{RLLR}^*) \\
& \quad + Re(\lambda_{LRRL}\lambda_{RLRL}^*)Re(A_{LRRL}A_{RLRL}^*))\} \\
& +M_8(s) \{|A_{LRLR}|^2 + |A_{RLLR}|^2 + |A_{LRRL}|^2 + |A_{RLRL}|^2\} \\
& +M_9(s) \{16|A_T|^2 + 64|A_{TE}|^2\}. \tag{13}
\end{aligned}$$

Here, we omitted λ_u because it is very small.

We will analyze the partially integrated CP asymmetry defined by Eq.(7) and examine its sensitivity to each Wilson coefficient. For numerical estimation, we set $(\rho, \eta) = (0.16, 0.33)$. At first, we investigate vector, scalar and tensor-type interactions, which are collectively *new local interactions*. The results of Ref. [17] make us predict the sensitivity of the CP asymmetry to each Wilson coefficient. The branching ratio is the most sensitive to the vector-type interactions, specially C_{LL} , and the contribution due to C_{RL} and C_{RR} it positive. And only the C_{LL} and C_{LR} have the weak and the strong phase, so we can expect that only the two types of interactions can make CP asymmetry be large, specially we can expect that the CP asymmetry is sizable by appropriate C_{LL} . However, C_{RL} and C_{RR} would suppress the CP asymmetry. The scalar and tensor-type interactions hardly interfere with each other or a vector-type interaction in the massless lepton limit. Thus, if a scalar or tensor-type interaction enters into our decay mode, it would suppress the CP asymmetry. In Figures 1 - 2, I plotted the correlation between the branching ratio and the CP asymmetry when C_{LL} or C_{LR} moves. Because the flow of each interaction depends on the type of the interaction, we can pin down the type of interaction which contributes to the processes once we measured these observable. These show behavior as expected in the above discussion. We should take attention to Figure 1, which shows the CP asymmetry can get much larger as the branching ratio is about predicted by the SM. It is because the partially integrated CP asymmetry for the SM is so suppressed why it is enlarged by 10^2 . Extremely, for $\phi_{LL} = \pi/4, \pi/2$ or $3\pi/4$, the asymmetry is the most enlarged when $A_{LL} \sim -1.2|C_{10}|, 0$ or $1.1|C_{10}|$. If we ignore the SM CP-violating contribution, A_9 and λ_u , C_{LL} enters into the asymmetry as followings

$$\frac{2 \int ds (Im(M_2(B_9 - C_{10}) - 2M_4C_7^{eff})(A_9^* + A_{LL})) \sin \phi_{LL}}{2m_b^8 \mathcal{B}_{SM} + \int ds M_2 |A_9 + A_{LL}|^2 + 2 \int ds Re(M_2(B_9 - C_{10}) - 2M_4C_7^{eff})(A_9^* + A_{LL})) \cos \phi_{LL}}, \tag{14}$$

where $M_2(s)$ and $M_4(s)$ are shown in Appendix A, and $2m_b^8 \mathcal{B}_{SM} \sim 0.72$. By choosing an approximate set of A_{LL} and ϕ_{LL} to hold

$$\int ds M_2 A_{LL} \sim -2 \int ds M_2 Re(B_9 - C_{10}) \cos \phi_{LL},$$

the asymmetry can get 10^{-1} . That is, if there is new physics through C_{LL} with weak phase, we have the possibility that we may pin down this type of interactions at the B factory in near future even if there is no contradiction with present experiments. And, Eq.(14) shows the correlation is very sensitive to whether ϕ_{LL} is infinitesimal or not. Thus the SM prediction point is far from other lines. In the same way, some A_{LR} and ϕ_{LR} enlarge the asymmetry and sensitive to ϕ_{LR} but, because $B_9 + C_{10} \ll B_9 - C_{10}$, its contribution is smaller than A_{LL} and ϕ_{LL} . And, in order to see how much the coefficient A_{LR} contribute to the asymmetry, we check at which the absolute value of the asymmetry becomes the maximum. By analogy with the analysis for A_{LL} and Eq.(14), we find that it has the largest value when, roughly,

$$2 \int ds M_2 Re(A_9) \sim - \int ds M_2 A_{LR},$$

numerically $A_{LR} \sim -1.4|C_{10}|$ or $-1.5|C_{10}|$ for $\phi_{LR} = \pi/4$ or $\pi/2$ and $3\pi/4$. (Note we ignored the term including $M_2 Re(B_9 + C_{10}) - 2M_4C_7^{eff}$ because it is much smaller than the remain in the dominator.)

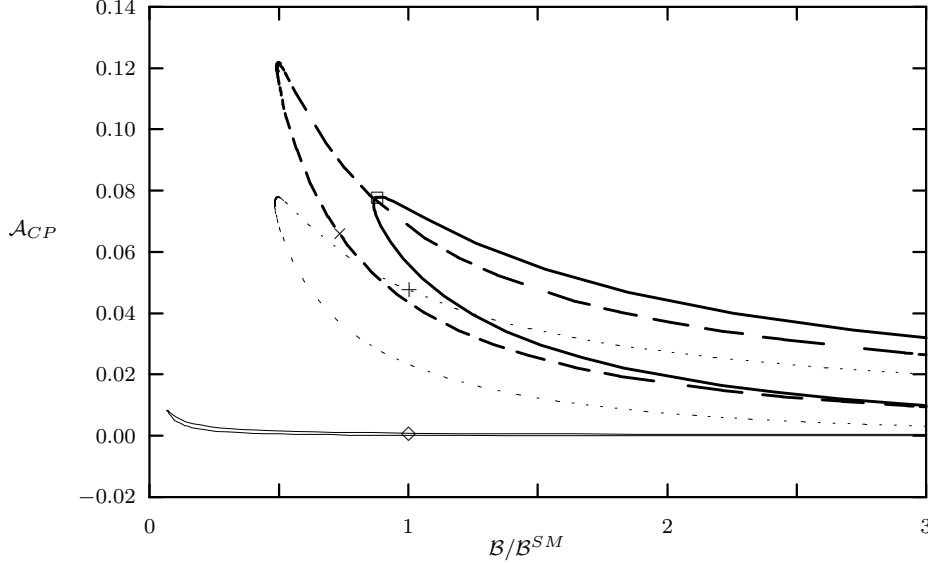


Figure 1: The correlation between $\mathcal{B}/\mathcal{B}^{SM}$ and \mathcal{A}_{CP} as A_{LL} moves, and $\phi_{LL} = 0$ (thin solid line), $\pi/4$ (dotted line), $\pi/2$ (thick solid line) and $3\pi/4$ (dashed line). The marks \diamond , $+$, \square and \times show the prediction for $\phi_{LL} = 0, \pi/4, \pi/2$ and $3\pi/4$ with $A_{LL} = 0$.

For C_{RL} and C_{RR} , the terms from $M_2 C_{RL}^2$ and $M_2 C_{RR}^2$ disappear in the numerator, so that the other terms,

$$-M_6(s)(C_9^{eff} - C_{10})C_{RL}^*, \quad (15)$$

$$-M_6(s)(C_9^{eff} + C_{10})C_{RL}^*, \quad (16)$$

which we ignored when we had discussed the sensitivity of C_{RL} and C_{RR} to the branching ratio, give significant effect to the CP asymmetry, so that the asymmetry may depend on ϕ_{RL} and ϕ_{RR} . Here, $M_6(s)$ is given in Appendix A. Eqs.(15) and (16) give similar contribution to the asymmetry except that it includes not M_2 but M_6 . Since $M_6 \ll M_2$ due to strange quark mass m_s , its sensitivity is tiny. We can also consider the correlation where A_{RL} and A_{RR} are very small strong phase, that is,

$$A_{RL} = A_9 + A'_{RL}, \quad (17)$$

$$A_{RR} = A_9 + A'_{RR}, \quad (18)$$

$$(19)$$

where A'_{RL} and A'_{RR} are real. In this case, the sign of the imaginary part of the $(B_9 - C_{10})(A_9 + A'_{RL})$ and $(B_9 + C_{10})(A_9 + A'_{RR})$ yields the difference between the correlations, however, the sensitivity is still tiny.

For scalar and tensor interactions, in the massless lepton limit, their Wilson coefficients appear only through the squared absolute. So, the asymmetry is almost independent of ϕ_S ($S = LLLR, LLRL, RLLR, RLRL$), ϕ_T and ϕ_{TE} and it gets only more suppressed as A_S , A_T or A_{TE} gets larger. Moreover, the sensitivity is very small because the corresponding kinematic functions include a factor m_l .

Next, consider only C_{BR} and C_{SL} , which is constrained by Eq.(10). Generally, without strong phase, these coefficients are expressed by

$$C_{BR} = A_{BR}e^{i\phi_{BR}}, \quad C_{SL} = A_{SL}e^{i\phi_{SL}}, \quad (20)$$

where ϕ_{BR} and ϕ_{SL} are independent weak phases. As shown in Ref. [18], the partially integrated branching ratio \mathcal{B} is more sensitive to C_{BR} than $C_{SL}^N \equiv (m_b/m_s)C_{SL}$ because of the strange quark mass m_s . This is true for the partially integrated CP asymmetry \mathcal{A}_{CP} . In other words, it is almost independent

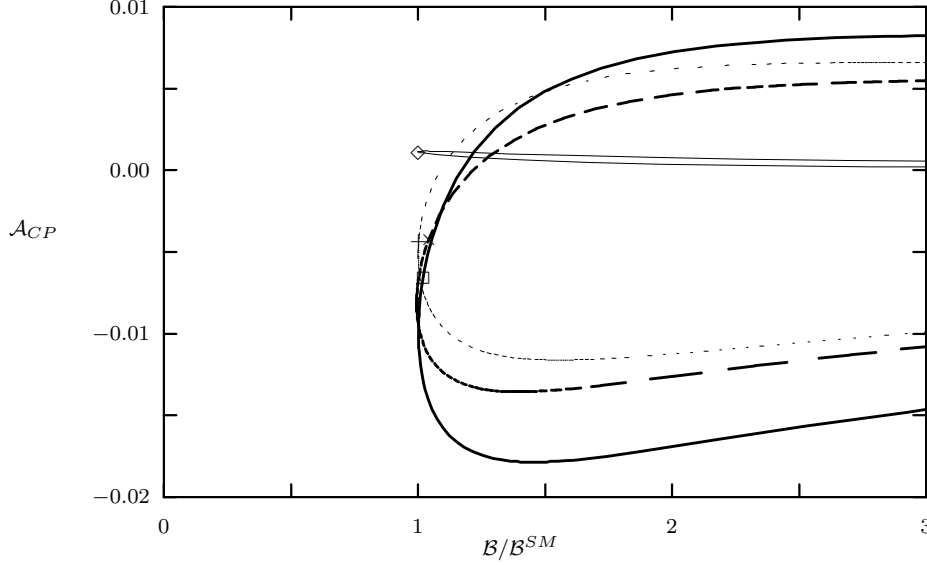


Figure 2: The correlation between $\mathcal{B}/\mathcal{B}^{SM}$ and \mathcal{A}_{CP} as A_{LR} moves, and $\phi_{LR} = 0$ (thin solid line), $\pi/4$ (dotted line), $\pi/2$ (thick solid line) and $3\pi/4$ (dashed line). The mark \diamond shows the standard model prediction. The marks \diamond , $+$, \square and \times show the prediction for $\phi_{LR} = 0, \pi/4, \pi/2$ and $3\pi/4$ with $A_{LR} = 0$.

of the phase ϕ_{SL} in comparison with ϕ_{BR} . And the asymmetry cannot be so enlarged by A_{SL} (or A_{BR}) with $\phi_{BR} = 0$. We can find this feature by comparing Figure 3 with Figure 4. In the former, we set ϕ_{SL} to 0, in the latter, however, we set $\phi_{SL} = \phi_{BR} \equiv \phi_{NL}$. By contrast with C_{SL} , the form of the correlation depends on C_{BR} considerably. Ignoring the SM contribution, in the case of $\phi_{SL} = \phi_{BR} = \phi_{NL}$, the asymmetry takes a form of

$$\frac{8m_b C_7^{eff} (m_s \int ds S_3 \cos \theta \text{Im}(B_9) + m_b \int ds S_4 \sin \theta \text{Im}(B_9)) \sin \phi_{NL}}{2m_b^8 \mathcal{B}_{NL} + 2m_b^8 \mathcal{B}_L - 8m_b C_7^{eff} (m_s \int ds S_3 \cos \theta \text{Re}(B_9) + m_b \int ds \sin \theta S_4 \text{Re}(B_9)) \cos \phi_{NL}}, \quad (21)$$

where \mathcal{B}_{NL} and \mathcal{B}_L are the partially integrated branching ratios. For the former, only non-vanishing new Wilson coefficients are A_{BR} and the latter has A_{SL} and $A_{BR} = A_{SL} = 0$. We set $\tan \theta = A_{BR}/A_{SL}^N$ and ignored the higher order terms about m_s/m_b . The definition of S_3 and S_4 is given in Appendix A. Since $\text{Im}(B_9) \ll \text{Re}(B_9)$, the partially integrated branching ratio is expressed by

$$\frac{1}{2m_b^8} \mathcal{B}_0 \left[\mathcal{B}_{NL} + \mathcal{B}_L - 8m_b C_7^{eff} \left(m_s \int ds S_3 \text{Re}(B_9) \cos \theta + m_b \int ds S_4 \text{Re}(B_9) \sin \theta \right) \cos \phi_{NL} \right]. \quad (22)$$

Eqs.(21) and (22) show that, when ϕ_{NL} rounds from 0 to 2π , the ellipse of the correlation, as shown in Figure 4, does. The size of A_{BR} , and also A_{SL} , is not so significant to enlarge the partially integrated CP asymmetry. Thus, these two types of interaction do not give so great influence to the partially integrated CP asymmetry even if there is another type of new interactions, say C_{LL} . For example, when we set $\phi_{LL} = \pi/2$ and we check the dependency of ϕ_{NL} on the asymmetry, it does not so largely changes the form of the correlation between \mathcal{B} and \mathcal{A} as A_{LL} moves negligible, as shown in Figure 5, so we must note if and only if very minute experiments were done.

3 Summary

I presented the model-independent analysis of the partially integrated CP asymmetry of the inclusive rare B decay $B \rightarrow X_s l^+ l^-$. CP violation is one of the most interesting topics to search new physics and understand baryogenesis at early universe, and many researchers has studied this observable through the both of experimental and theoretical approaches. The process $B \rightarrow X_s l^+ l^-$ is experimentally clean, and

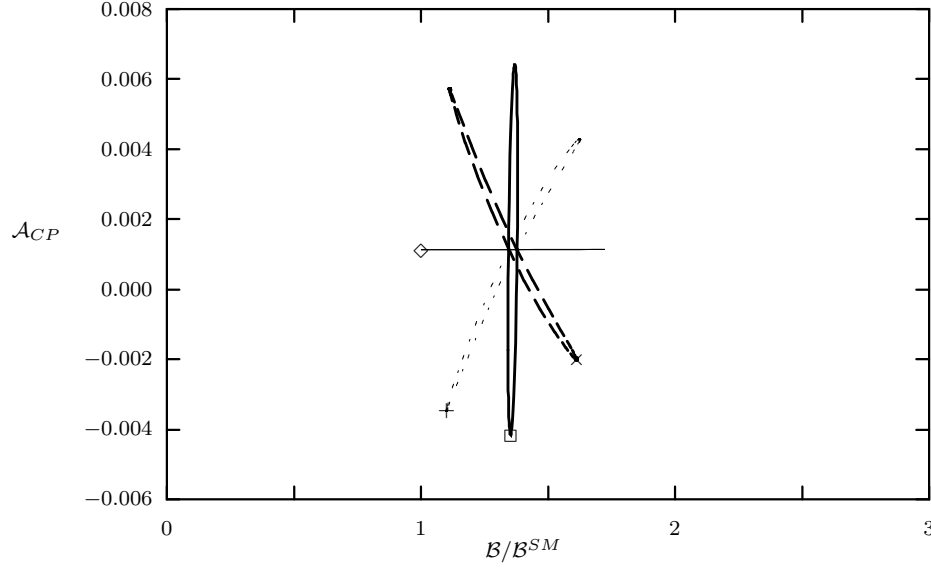


Figure 3: The correlation between $\mathcal{B}/\mathcal{B}^{SM}$ and \mathcal{A}_{CP} as θ moves, and $\phi_{BR} = 0$ (thin solid line), $\pi/4$ (dotted line), $\pi/2$ (thick solid line) and $3\pi/4$ (dashed line), where $\tan \theta = A_{BR}/A_{SL}^N$. We set $\phi_{SL} = 0$. And, for $A_{SL} = -2C_7^{eff}$ and $A_{BR} = -2C_7^{eff}$, plotted some marks, \diamond ($\phi_{BR} = 0$), $+$ ($\phi_{BR} = \pi/4$), \square ($\phi_{BR} = \pi/2$) and \times ($\phi_{BR} = 3\pi/4$).

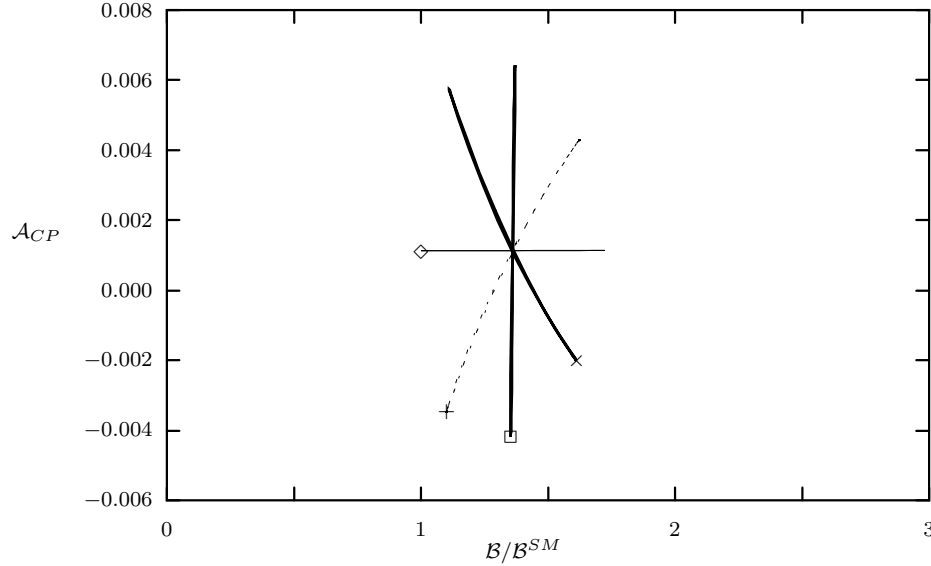


Figure 4: The correlation between $\mathcal{B}/\mathcal{B}^{SM}$ and \mathcal{A}_{CP} as θ moves, and $\phi_{NL} = 0$ (thin solid line), $\pi/4$ (thin dotted line), $\pi/2$ (thick solid line) and $3\pi/4$ (thick solid line), where $\tan \theta = A_{BR}/A_{SM}^N$. We set $\phi_{NL} \equiv \phi_{SL} = \phi_{BR}$. And, for $A_{SL} = -2C_7^{eff}$ and $A_{BR} = -2C_7^{eff}$, I plotted some marks, \diamond ($\phi_{NL} = 0$), $+$ ($\phi_{NL} = \pi/4$), \square ($\phi_{NL} = \pi/2$) and \times ($\phi_{NL} = 3\pi/4$).

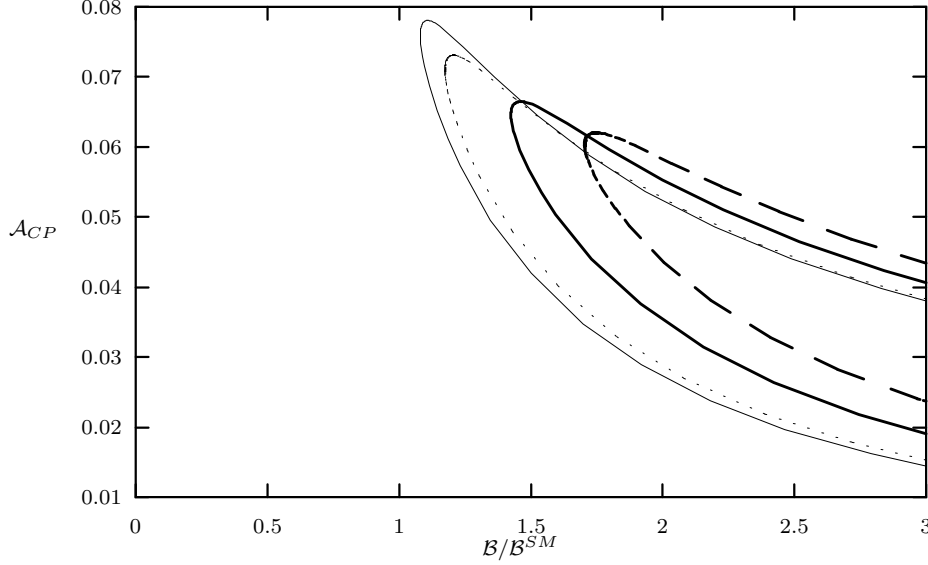


Figure 5: The correlation of $\mathcal{B}/\mathcal{B}_0$ and $\mathcal{A}_{CP}/\mathcal{A}_{CP}^{SM}$ as A_{LL} moves for $\phi_{LL} = \pi/2$ and $\phi_{NL} = 0$ (thin solid line), $\pi/4$ (dotted line), $\pi/2$ (thick solid line) and $3\pi/4$ (dashed line). Here the definition of ϕ_{NL} is the same as Figure 4.

there is a possibility that this mode is found by KEKB and PEP-II B factories. Because $B \rightarrow X_s l^+ l^-$ is flavor changing neutral current (FCNC) process, it is the most sensitive to the various extensions of the standard model (SM). Our analysis includes the full operator basis, that is, twelve independent four Fermi operators. In the SM, only three type Wilson coefficients contribute to $B \rightarrow X_s l^+ l^-$, and the partially integrated CP asymmetry has order of 10^{-3} . We investigated the correlation of the partially integrated branching ratio and the partially integrated CP asymmetry, and then can conclude that only C_{LL} , the coefficient of the operator $(\bar{s}_L \gamma_\mu b_L \bar{l}_L \gamma^\mu l_L)$, can give meaningful contribution to our process. This cause is the same as the branching ratio[17], that is, the large interference between $(B_9 - C_{10})$ and C_{LL} . Since $(B_9 + C_{10}) \ll (B_9 - C_{10})$, the contribution of C_{LR} , the coefficient of operator $(\bar{s}_L \gamma_\mu b_L \bar{l}_R \gamma^\mu l_R)$, is less than C_{LL} . However, the Wilson coefficients of the other new local interactions beyond SM work only to suppress the asymmetry, because we assumed there was no new strong phase, and then they have no interference with the SM interactions. In order to contrast with the left-right symmetric model, we made C_{RL} and C_{RR} have very small strong phase, however it changes the size of the CP asymmetry a little. For C_{BR} and C_{SL} , the coefficients of the $B \rightarrow X_s \gamma$ operators, although the asymmetry depends on the weak phase ϕ_{BR} of C_{BR} largely, their size gives little contribution to the asymmetry. Thus, the dependency of two coefficients is much smaller than that of A_{LL} . Note that the branching ratio also depends on the ϕ_{BR} .

Our analysis contains the special cases like the MSSM and the 2HDM. In the MSSM, a special case is $C_{BR} = C_{SL} = 2C_7$, $C_{LL} = C_9^{eff} - C_{10}$ and $C_{LR} = C_9^{eff} + C_{10}$. This is just expressed as an example by Figure 4. Therefore, the asymmetry is very suppressed like the standard model, although the branching ratio can be large or not. However, the model has the possibility of the conversion of sign of C_{10} . In this case, Figures 1 and 2 show that the CP asymmetry may be enlarged. Large contributions to A_{CP} was pointed out by Ref. [10]. Figure 4 includes the rough character of 2HDM, where a new weak phase enters into C_{BR} and C_{SL} with the deviation from the SM prediction for the numerical values of C_{BR} , C_{SL} , C_{LL} and C_{LR} . When $\sin \phi_{NL}$ is small, the asymmetry is suppressed, however when $\sin \phi_{NL}$ is close to unit, it change with the sign of C_{BR} [14]. Once the CP asymmetry is measured, we will be able to constrain the extended models by comparing the data with our numerical analysis. If we will get the signature of the asymmetry in it, we can conclude that there is a new $(V - A) \otimes (V - A)$ interaction and / or sizable strong coupling. Otherwise, the analysis of the present paper cannot constrain us within some models, so we have to wait the future experiments to get some informations on the CP from $B \rightarrow X_s l^+ l^-$.

I would like to thank to C.S. Kim, T. Yoshikawa, and T. Morozumi who give suggestion and comments.

Appendix

A Kinematic Functions

We list a set of the kinematic functions, which decide the behavior of the branching ratio and the CP asymmetry for the decay $b \rightarrow sl^+l^-$, and show the general expression of the direct CP asymmetry. The ratio is shown by Eq.(2). We follow Refs.[17, 25] as the notation. That is, the functions are given by

$$\begin{aligned}
S_1(s) &= -\frac{4}{s}u(s)\{s^2 - \frac{1}{3}u(s)^2 - (m_b^2 - m_s^2)^2\}, \\
S_2(s) &= -16u(s)m_b m_s, \\
S_3(s) &= 4u(s)(s + m_b^2 - m_s^2), \\
S_4(s) &= 4u(s)(s - m_b^2 + m_s^2), \\
M_1(s) &= (m_s^2 + m_b^2)S_1(s) + 2m_b m_s S_2(s), \\
M_2(s) &= 2u(s)(-\frac{1}{3}u(s)^2 - s^2 + (m_b^2 - m_s^2)^2), \\
M_4(s) &= m_s^2 S_3(s) + m_b^2 S_4(s) \\
M_6(s) &= m_b m_s (S_3(s) + S_4(s)), \\
M_8(s) &= 2u(s)(m_b^2 + m_s^2 - s)s, \\
M_9(s) &= 2u(s)\{-\frac{2}{3}u(s)^2 - 2(m_b^2 + m_s^2)s + 2(m_b^2 - m_s^2)^2\}, \tag{23}
\end{aligned}$$

where we neglected lepton mass.

With the above functions, we can express the partially integrated CP asymmetry delivered from the matrix element Eq.(1), that is,

$$\mathcal{A}_{CP} \equiv \frac{\int_{1\text{GeV}^2}^8 ds (d\mathcal{N}_{CP}(s)/ds)}{\int_{1\text{GeV}^2}^8 ds (d\mathcal{D}_{CP}(s)/ds)} \equiv \frac{\mathcal{N}_{CP}}{\mathcal{D}_{CP}}, \tag{24}$$

where

$$\begin{aligned}
\frac{d\mathcal{N}_{CP}(s)}{ds} &= -\frac{1}{m_b^8} \mathcal{B}_0[\\
&S_1(s) \{2m_s^2 \text{Im}(\lambda_{SL})\text{Im}(B_{SL}A_{SL}^*) + 2m_b^2 \text{Im}(\lambda_{BR})\text{Im}(B_{BR}A_{BR}^*)\} \\
&+ S_2(s) \{2m_b m_s (\text{Im}(\lambda_{SL})\text{Im}(A_{SL}B_{BR}^*) + \text{Im}(\lambda_{BR})\text{Im}(B_{SL}^*A_{BR}) + \text{Im}(\lambda_{SL}\lambda_{BR}^*)\text{Im}(A_{SL}A_{BR}^*))\} \\
&+ S_3(s) \{2m_s^2 (\text{Im}(\lambda_{SL})\text{Im}(A_{SL}B_{LL}^*) + \text{Im}(\lambda_{LL})\text{Im}(B_{SL}^*A_{LL}) + \text{Im}(\lambda_{SL}\lambda_{LL}^*)\text{Im}(A_{SL}A_{LL}^*) \\
&\quad + \text{Im}(\lambda_{SL})\text{Im}(A_{SL}B_{LR}^*) + \text{Im}(\lambda_{LR})\text{Im}(B_{SL}^*A_{LR}) + \text{Im}(\lambda_{SL}\lambda_{LR}^*)\text{Im}(A_{SL}A_{LR}^*)) \\
&\quad + 2m_b m_s (\text{Im}(\lambda_{BR})\text{Im}(A_{BR}B_{RL}^*) + \text{Im}(\lambda_{RL})\text{Im}(B_{BR}^*A_{RL}) + \text{Im}(\lambda_{BR}\lambda_{RL}^*)\text{Im}(A_{BR}A_{RL}^*) \\
&\quad + \text{Im}(\lambda_{BR})\text{Im}(A_{BR}B_{RR}^*) + \text{Im}(\lambda_{RR})\text{Im}(B_{BR}^*A_{RR}) + \text{Im}(\lambda_{BR}\lambda_{RR}^*)\text{Im}(A_{BR}A_{RR}^*))\} \\
&+ S_4(s) \{2m_b^2 (\text{Im}(\lambda_{BR})\text{Im}(A_{BR}B_{LL}^*) + \text{Im}(\lambda_{LL})\text{Im}(B_{BR}^*A_{LL}) + \text{Im}(\lambda_{BR}\lambda_{LL}^*)\text{Im}(A_{BR}A_{LL}^*) \\
&\quad + \text{Im}(\lambda_{BR})\text{Im}(A_{BR}B_{LR}^*) + \text{Im}(\lambda_{LR})\text{Im}(B_{BR}^*A_{LR}) + \text{Im}(\lambda_{BR}\lambda_{LR}^*)\text{Im}(A_{BR}A_{LR}^*)) \\
&\quad + 2m_b m_s (\text{Im}(\lambda_{SL})\text{Im}(A_{SL}B_{RL}^*) + \text{Im}(\lambda_{RL})\text{Im}(B_{SL}^*A_{RL}) + \text{Im}(\lambda_{SL}\lambda_{RL}^*)\text{Im}(A_{SL}A_{RL}^*) \\
&\quad + \text{Im}(\lambda_{SL})\text{Im}(A_{SL}B_{RR}^*) + \text{Im}(\lambda_{RR})\text{Im}(B_{SL}^*A_{RR}) + \text{Im}(\lambda_{SL}\lambda_{RR}^*)\text{Im}(A_{SL}A_{RR}^*))\} \\
&+ M_2(s) \{2 (\text{Im}(\lambda_{LL})\text{Im}(B_{LL}A_{LL}^*) + \text{Im}(\lambda_{LR})\text{Im}(B_{LR}A_{LR}^*) \\
&\quad + \text{Im}(\lambda_{RL})\text{Im}(B_{RL}A_{RL}^*) + \text{Im}(\lambda_{RR})\text{Im}(B_{RR}A_{RR}^*))\} \\
&+ M_6(s) \{-2 (\text{Im}(\lambda_{LL})\text{Im}(A_{LL}B_{RL}^*) + \text{Im}(\lambda_{RL})\text{Im}(B_{LL}^*A_{RL}) + \text{Im}(\lambda_{LL}\lambda_{RL}^*)\text{Im}(A_{LL}A_{RL}^*) \\
&\quad + \text{Im}(\lambda_{LR})\text{Im}(A_{LR}B_{RR}^*) + \text{Im}(\lambda_{RR})\text{Im}(B_{LR}^*A_{RR}) + \text{Im}(\lambda_{LR}\lambda_{RR}^*)\text{Im}(A_{LR}A_{RR}^*) \\
&\quad + (\text{Im}(\lambda_{LRLR})\text{Im}(A_{LRLR}B_{RLLR}^*) + \text{Im}(\lambda_{RLLR})\text{Im}(B_{LRLR}^*A_{RLLR}))\}
\end{aligned}$$

$$\begin{aligned}
& +Im(\lambda_{LRLR}\lambda_{RLLR}^*)Im(A_{LRLR}A_{RLLR}^*) \\
& +Im(\lambda_{LRRL})Im(A_{LRRL}B_{RLRL}^*) + Im(\lambda_{RLRL})Im(B_{LRRL}^*A_{RLRL}) \\
& +Im(\lambda_{LRRL}\lambda_{RLRL}^*)Im(A_{LRRL}A_{RLRL}^*)\} \\
& +M_8(s) \{2(Im(\lambda_{LRLR})Im(B_{LRLR}A_{LRLR}^*) + Im(\lambda_{RLLR})Im(B_{RLLR}A_{RLLR}^*)) \\
& +Im(\lambda_{LRRL})Im(B_{LRRL}A_{LRRL}^*) + Im(\lambda_{RLRL})Im(B_{RLRL}A_{RLRL}^*)\} \\
& +M_9(s) \{32Im(\lambda_T)Im(B_TA_T^*) + 128Im(\lambda_{TE})Im(B_{TE}A_{TE}^*)\}, \tag{25}
\end{aligned}$$

and

$$\begin{aligned}
\frac{d\mathcal{D}_{CP}(s)}{ds} &= 2 \left. \frac{d\mathcal{B}(s)}{ds} \right|_{C_{XX} \rightarrow B_{XX}} + \frac{1}{m_b^8} \mathcal{B}_0[\\
& S_1(s) \{m_s^2 (|A_{SL}|^2 + 2Re(\lambda_{SL})Re(B_{SL}A_{SL}^*)) + m_b^2 (|A_{BR}|^2 + 2Re(\lambda_{BR})Re(B_{BR}A_{BR}^*))\} \\
& +S_2(s) \{2m_b m_s (Re(\lambda_{SL})Re(A_{SL}B_{BR}^*) + Re(\lambda_{BR})Re(B_{SL}^*A_{BR}) + Re(\lambda_{SL}\lambda_{BR}^*)Re(A_{SL}A_{BR}^*))\} \\
& +S_3(s) \{2m_s^2 (Re(\lambda_{SL})Re(A_{SL}B_{LL}^*) + Re(\lambda_{LL})Re(B_{SL}^*A_{LL}) + Re(\lambda_{SL}\lambda_{LL}^*)Re(A_{SL}A_{LL}^*) \\
& +Re(\lambda_{SL})Re(A_{SL}B_{LR}^*) + Re(\lambda_{LR})Re(B_{SL}^*A_{LR}) + Re(\lambda_{SL}\lambda_{LR}^*)Re(A_{SL}A_{LR}^*)) \\
& +2m_b m_s (Re(\lambda_{BR})Re(A_{BR}B_{RL}^*) + Re(\lambda_{RL})Re(B_{BR}^*A_{RL}) + Re(\lambda_{BR}\lambda_{RL}^*)Re(A_{BR}A_{RL}^*) \\
& +Re(\lambda_{BR})Re(A_{BR}B_{RR}^*) + Re(\lambda_{RR})Re(B_{BR}^*A_{RR}) + Re(\lambda_{BR}\lambda_{RR}^*)Re(A_{BR}A_{RR}^*))\} \\
& +S_4(s) \{2m_b^2 (Re(\lambda_{BR})Re(A_{BR}B_{LL}^*) + Re(\lambda_{LL})Re(B_{BR}^*A_{LL}) + Re(\lambda_{BR}\lambda_{LL}^*)Re(A_{BR}A_{LL}^*) \\
& +Re(\lambda_{BR})Re(A_{BR}B_{LR}^*) + Re(\lambda_{LR})Re(B_{BR}^*A_{LR}) + Re(\lambda_{BR}\lambda_{LR}^*)Re(A_{BR}A_{LR}^*)) \\
& +2m_b m_s (Re(\lambda_{SL})Re(A_{SL}B_{RL}^*) + Re(\lambda_{RL})Re(B_{SL}^*A_{RL}) + Re(\lambda_{SL}\lambda_{RL}^*)Re(A_{SL}A_{RL}^*) \\
& +Re(\lambda_{SL})Re(A_{SL}B_{RR}^*) + Re(\lambda_{RR})Re(B_{SL}^*A_{RR}) + Re(\lambda_{SL}\lambda_{RR}^*)Re(A_{SL}A_{RR}^*))\} \\
& +M_2(s) \{|A_{LL}|^2 + 2Re(\lambda_{LL})Re(B_{LL}A_{LL}^*) + |A_{LR}|^2 + 2Re(\lambda_{LR})Re(B_{LR}A_{LR}^*) \\
& +|A_{RL}|^2 + 2Re(\lambda_{RL})Re(B_{RL}A_{RL}^*) + |A_{RR}|^2 + 2Re(\lambda_{RR})Re(B_{RR}A_{RR}^*)\} \\
& +M_6(s) \{-2(Re(\lambda_{LL})Re(A_{LL}B_{RL}^*) + Re(\lambda_{RL})Re(B_{LL}^*A_{RL}) + Re(\lambda_{LL}\lambda_{RL}^*)Re(A_{LL}A_{RL}^*) \\
& +Re(\lambda_{LR})Re(A_{LR}B_{RR}^*) + Re(\lambda_{RR})Re(B_{LR}^*A_{RR}) + Re(\lambda_{LR}\lambda_{RR}^*)Re(A_{LR}A_{RR}^*)) \\
& + (Re(\lambda_{LRLR})Re(A_{LRLR}B_{RLLR}^*) + Re(\lambda_{RLLR})Re(B_{LRLR}^*A_{RLLR})) \\
& +Re(\lambda_{LRLR}\lambda_{RLLR}^*)Re(A_{LRLR}A_{RLLR}^*) \\
& +Re(\lambda_{LRRL})Re(A_{LRRL}B_{RLRL}^*) + Re(\lambda_{RLRL})Re(B_{LRRL}^*A_{RLRL}) \\
& +Re(\lambda_{LRRL}\lambda_{RLRL}^*)Re(A_{LRRL}A_{RLRL}^*)\} \\
& +M_8(s) \{|A_{LRLR}|^2 + 2Re(\lambda_{LRLR})Re(B_{LRLR}A_{LRLR}^*) + |A_{RLLR}|^2 + 2Re(\lambda_{RLLR})Re(B_{RLLR}A_{RLLR}^*) \\
& +|A_{LRRL}|^2 + 2Re(\lambda_{LRRL})Re(B_{LRRL}A_{LRRL}^*) + |A_{RLRL}|^2 + 2Re(\lambda_{RLRL})Re(B_{RLRL}A_{RLRL}^*)\} \\
& +M_9(s) \{16(|A_T|^2 + 2Re(\lambda_T)Re(B_TA_T^*)) + 64(|A_{TE}|^2 + 2Re(\lambda_{TE})Re(B_{TE}A_{TE}^*))\}. \tag{26}
\end{aligned}$$

The first term $d\mathcal{B}/ds|_{C_{XX} \rightarrow B_{XX}}$ in Eq.(26) is the differential branching ratio given by Eq.(2) after replacing all Wilson coefficients C_{XX} with B_{XX} , respectively.

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